PUNJAB PUBLIC SERVICE COMMISSION

COMBINED COMPETITIVE EXAMINATION
FOR RECRUITMENT TO THE POSTS OF
PROVINCIAL MANAGEMENT SERVICE, ETC -2023
CASE NO. 1C2024

SUBJECT: MATHEMATICS (PAPER-I)

TIME ALLOWED: THREE HOURS   **MAXIMUM MARKS: 100**


NOTE:
   i.   All the parts (if any) of each Question must be attempted at one place instead of at different places.
   ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
   iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
   iv. Extra attempt of any question or any part of the question will not be considered.

NOTE: **Attempt FIVE Questions in All including THREE questions from Part-A and TWO questions from Part-B. Calculator is allowed. (Non-Programmable)**

PART-A

Q.No.1   **(a)** For the function $f(x)$ graphed here, find the following limits or explain why they do not exist.

   (i) $\lim\limits_{x \to 1} f(x)$,   (ii) $\lim\limits_{x \to 2} f(x)$,

   (iii) $\lim\limits_{x \to 2.5} f(x)$,   (iv) $\lim\limits_{x \to 2.5} f(x)$
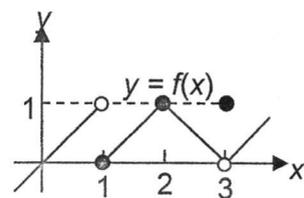
(b) Find the values of a and b that make the following function differentiable for all x-values.

$$f(x) = \begin{cases} ax + b, & x > -1 \\ bx^2 - 3, & x \le -1 \end{cases}$$

(10+10=20 Marks)

Q.No.2   **(a)** Discuss the validity of Rolle's theorem of $f(x) = x(x+3)e^{-\frac{x}{2}}$ on $[-3, 0]$. Find 'c' (if possible).

(b) Use Mean Value Theorem to show that $|\sin x - \sin y| \le |x - y|$ for any real numbers x, y.

(10+10=20 Marks)

Q.No.3   **(a)** Find the area of region bounded by the curve $y = x^2 - 4x$, the x-axis, and the lines $x = 1$ and $x = 3$.

(b) Using rectangular rule for $n = 5$, approximate the value of the definite integral $\int\limits_0^1 \frac{dx}{1 + x^3}$.

(10+10=20 Marks)

Q.No.4   **(a)** Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where a, b, c are positive.

(b) The area in the first quadrant bounded by the parabola $y^2 = 4ax$ and its latus rectum is revolved about the x-axis. Find the volume of the solid generated.

(10+10=20 Marks)

Q.No.5   **(a)** Solve $x\frac{dy}{dx} + y = y^2 \ln x$.

(b) An oscillator moves under the forces:
   restorative force $= -kx$
   damping force $= -2\mu\dot{x}$
   driving force $= F_0 e^{-at}$ each force being per unit mass.
   Set up and solve the equation of motion completely.

(10+10=20 Marks)

PART-B

Q.No.6 **(a)** Prove that the function $f(z) = \sqrt{|xy|}$ is not differentiable at origin although Cauchy-Riemann conditions are satisfied at origin.

(b) Evaluate $\int_C \dfrac{z}{(z-1)(z+2i)} dz$ where (i) $C : |z| = \dfrac{1}{2}$, (ii) $C : |z| = \dfrac{3}{2}$.

(10+10=20 Marks)

Q.No.7 **(a)** Transform $x^2 + y^2 - z = 9$ into spherical coordinates.

(b) The tangent at any point on the curve $x^3 + y^3 = 2a^3$ makes intercepts p and q on the coordinate axes respectively. Show that $p^{-3/2} + q^{-3/2} = 2^{-1/2} a^{-3/2}$

(10+10=20 Marks)

Q.No.8 **(a)** Find the tangent line and normal plane to the curve $\vec{x} = t\,\hat{e}_1 + t^2\,\hat{e}_2 + t^3\,\hat{e}_3$ at $t = 1$.

(b) Find the curvature and torsion of $\vec{r} = (a\cos\theta,\ a\sin\theta,\ a\theta\cot\alpha)$.

(10+10=20 Marks)

ROLL NO

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CASE NO. 1C2024

SUBJECT: MATHEMATICS (PAPER-II)

TIME ALLOWED: THREE HOURS   MAXIMUM MARKS: 100

NOTE:
 i. All the parts (if any) of each Question must be attempted at one place instead of at different places.
 ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
 iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
 iv. Extra attempt of any question or any part of the question will not be considered.

NOTE: **Attempt any FIVE questions. Calculator is allowed. (Non-Programmable)**

Q.No.1   **(a)** Show that if every element of a group G is its own inverse, then G is an Abelian group.

(b) In a symmetric group of degree 3, give an example of two elements x, y such that $(x.y)^2 \neq x^2.y^2$ **(10+10=20 Marks)**

Q.No.2   **(a)** Show that normalizer of a subset H of a group G is a subgroup of G.

(b) Show that the centralizer of a subset H in a group G is a subgroup of G.   **(10+10=20 Marks)**

Q.No.3   **(a)** Define an integral domain. If p is a prime number, then show that ring of integers mod p is an integral domain.

(b) Let F be a field of real numbers. Then set of all real valued functions whose nth derivative exist for $n = 1, 2, \dots$, is a subspace of all real valued continuous function on $[0,1]$.   **(10+10=20 Marks)**

Q.No.4   **(a)** If G is a non-Abelian group of order 6, then show that G is isomorphic on to S_3.

(b) Let G_1 be the group of positive real numbers under multiplication and G_2 the group of all real numbers under addition. Define $\phi: G_1 \to G_2$ by $\phi(x) = \log_{10} x$. Show that ϕ is a bijective homomorphism.   **(10+10=20 Marks)**

Q.No.5   **(a)** Let X be any infinite set, and let the set J consist of the empty set ϕ together with all the subsets of X whose complements are finite. Show that J is a topology on X.

(b) Define a metrizable topological space. Give an example of a topological space which is not metrizable.   **(10+10=20 Marks)**

Q.No.6   **(a)** Let $(X, <.>)$ be an inner product space. Show that
$| < x, y > | \leq ||x|| ||y||$ for all $x, y \in X$, where the equality sign holds if and only if $\{x, y\}$ is a linearly dependent set.
(b) Show that the space $C[a, b]$ is not an inner product space   **(10+10=20 Marks)**

Q.No.7   **(a)** Show that the following can happen for 2x2 matrices A and B.
        (1) $A^2 = 0$ even though $A \neq 0$.
        (2) $AB \neq BA$.

(b) Suppose that $T: \mathbb{R}^4 \to \mathbb{R}^3$ is a linear transformation such that
$T([1, 0, -1, 2]^T) = [2, 1, 0]^T$ and $T([1, 1, -2, 0]^T) = [-1, 2, 4]^T$.
Compute $T([-2, -5, 7, 6])$.   **(10+10=20 Marks)**

Q.No.8   **(a)** Find the eigen values and the corresponding eigen vectors of $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

(b) Any two eigen vectors corresponding to two distinct eigen values of orthogonal matrix are orthogonal.   **(10+10=20 Marks)**